INDIAN SCHOOL AL WADI AL KABIR

Assessment - 1

Class: XI Sub: MATHEMATICS (041) Max Marks: 80

Date: 21.09.2023 Time: 3 hr

General Instructions:

1. This question paper is divided in to 6 sections- A, B, C, D and E
2. Section A comprises of 20 MCQ type questions of 1 mark each.
3. Section B comprises of 5 Very Short Answer Type Questions of 2 marks each.
4. Section C comprises of 6 Short Answer Type Questions of 3 marks each.
5. Section D comprises of 4 Long Answer Type Questions of 5 marks each.
6. Section E comprises of 3 source based / case based / passage-based questions (4 marks each) with sub parts.
7. Internal choice has been provided for certain questions

SECTION – A

(Each MCQ Carries 1 Mark)

1 Given U = [-5, 5] and A is (-3, 5], then A^C is

a) [-5, -3) b) (4, 5] c) [-5, -3] d) [4, 5]

2 For disjoint sets A and B, n(A) = 3 and n(B) = 5 then n(A ∩ B) is

a) 0 b) 3 c) 5 d) 8

3 A and B are two sets such that n(A – B) = 20 + ***x***, n(B - A) = 3***x*** and n(A ∩ B) = ***x*** + 1. If n(A) = n(B) then the value of ***x*** is

a) 3 b) 5 c) 8 d) 10

4 If $f(x) = \frac{2x}{1-x^2}$ then $f(\tan\theta)$ is

a) $\cos 2\theta$ b) $\sin 2\theta$ c) $\tan 2\theta$ d) $\cot 2\theta$

5 The Value of [4.97] is

a) 4.97 b) -4.97 c) 5 d) 4

6 Angle formed by the minute hand of a clock in 20 minutes is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{3\pi}{4}$ d) $\frac{2\pi}{3}$

7 $\tan\left(-\frac{11\pi}{6}\right)$ is equal to

a) $-\sqrt{3}$ b) $\sqrt{3}$ c) $\frac{1}{\sqrt{3}}$ d) $-\frac{1}{\sqrt{3}}$

8 The radian representation of $20^0\ 30^l$ is

a) $20.5\pi^c$ b) $\frac{41}{360}\pi^c$ c) $\frac{81}{360}\pi^c$ d) $\frac{121}{360}\pi^c$

9 The value of $\sin 15^0$

a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ b) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ c) $\frac{2\sqrt{2}}{1-\sqrt{3}}$ d) $\frac{2\sqrt{2}}{\sqrt{3}-1}$

10 The simplified form of i^{257} is

a) i b) $-i$ c) 1 d) -1

11 If $z_1 = 2 + 3i$ and $z_2 = -5i + 9$, then $\text{Re}(z_1 + z_2)$ is

a) – 3 b) 7 c) 11 d) 12

12 The absolute value of the complex number $z = 3 + 6i$

a) 3 b) 6 c) 9 d) $3\sqrt{5}$

13 The value of $\frac{i^{4n+3} - i^{4n-3}}{2}$ is

a) i b) $-i$ c) 1 d) -1

14 If $x < 5$, then which of the following is correct

a) $-x < -5$ b) $-x \leq -5$ c) $-x > -5$ d) $-x \geq -5$

15 In an experiment, a solution of hydrochloric acid is to be kept between 30° Celsius and 35° Celsius. What is the range of temperature in degree Fahrenheit if conversion is Celsius, $C = \frac{5}{9} \times (F - 32)$

a) 30F and 35F b) 54F and 63F c) 86F and 95F d) None of these

16 The number of ways in which five articles be put in four boxes is

a) 4^4 b) 4^5 c) 5^4 d) 5^5

17 If $\frac{1}{8!} + \frac{1}{9!} = \frac{x}{10!}$, then the value of x is

a) 64 b) 81 c) 100 d) None of these

18 There are 4 bus routes between A and B and 3 bus routes between B and C. A man can travel round the trip in number of ways by bus from A to C via B. If he does not to use a bus route more than once in how many ways can he make round trip.

a) 72 b) 24 c) 18 d) 12

Directions: In the following 2 questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is True

19 **Assertion (A):** $^{10}C_3 = 120$
Reason (R): $^{n}C_r = \frac{n!}{(n-r)!}$

a) b) c) d)

20 **Assertion (A):** The variance of 5, 5, 5, 5 is zero
Reason (R): Variance (σ^2) $= \frac{1}{n}\sum_{i=1}^{n}(xi - \bar{x})^2$

a) b) c) d)

SECTION – B

(Each Question Carries 2 Marks)

21 If $\tan\theta = \frac{1}{2}$ and $\tan\emptyset = \frac{1}{3}$, then find the value of $\theta + \emptyset$

- OR -

Prove that $\cot x \cdot \cot 2x - \cot 2x \cdot \cot 3x - \cot 3x \cdot \cot x = 1$

22 Express $\frac{3-i}{5+6i}$ in the form of (a + *i*b)

23 Solve the following system of linear inequalities:
$5x - 7 < 3(x + 3)$ & $1 - \frac{3x}{2} \leq x - 4$

24 How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of the digits is not allowed?

- OR -

In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

25 Find the mean deviation about the median for the data: 2, 3, 5, 6, 8, 10, 12, 17, 20, 26

SECTION – C

(Each Question Carries 3 Marks)

26 If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 3, 5}, B = {2, 4, 6, 7} and C = {2, 3, 4, 8} then find (i) $(B \cup C)^I$ and (ii) $(C - A)^I$

27 Find the domain and range of the function $\sqrt{x^2 - 4}$

28 Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

- OR -

Evaluate $\cos\left(\frac{3\pi}{2} + x\right) \cos(2\pi + x) \left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$

29 Evaluate $(1 + i)^6 + (1 - i)^3$

- OR -

If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, then find $x + y$

30 Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,
(i) do the words start with P
(ii) do all the vowels always occur together
(iii) do the vowels never occur together

- OR -

Find the Value of *'n'* such that ${}^nP_5 = 42.{}^nP_3$

31 Find the mean deviation about the mean for the following data:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of Students	4	6	10	20	10	6	4

SECTION – D

(Each Question Carries 5 Marks)

32 Let U = {x ∈ N : x ≤ 8}, A = $\{x \in N : 5 < x^2 < 50\}$,
B = {x ∈ N : x is a prime number less than 10}.

(i) Draw a Venn Diagram to show the relationship between the given sets
(ii) list the elements of A^I
(iii) list the elements of B^I
(iv) list the elements of A – B
(v) list the elements of $A \cap B^I$

33 Prove that: $\cos 2x \cdot \cos \frac{x}{2} - \cos 3x \cdot \cos \frac{9x}{2} = \sin 5x \cdot \sin \frac{5x}{2}$

- OR -

Prove that: $2\cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$

34 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(i) no girl?
(ii) at least one boy and one girl?
(iii) at least 3 girls?

- OR -

What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these

(i) four cards are of the same suit
(ii) four cards belong to four different suits
(iii) are face cards
(iv) two are red cards and two are black cards
(v) cards are of the same colour?

35 The diameters of circles (in mm) drawn in a design are given below. Calculate Mean, Variance and Standard Deviation for the data

Diameter	33 – 36	37 – 40	41 – 44	45 – 48	49 – 52
Number of Circles	15	17	21	22	25

SECTION – E

(CASE STUDY - Each Question Carries 4 Marks)

36 Vision of Infinity quiz was going on in ISWK. The 3rd round is activity round and each team will get 3 questions to answer, were 3rd question is with an optional question also. The team which completes the task first will get 30 points. The questions planned for the 3rd round as follows. Help your team to get the correct answers in the least amount of time.

(i) A and B are two sets such that n(A – B) = 20 + x, n(B – A) = 3x and n(A ∩ B) = x + 1. If n(A) = n(B) then find 'x'. (1m)

(ii) Check whether the following statement is True or False with reason:
A and B are two sets such that $n(A \cap \bar{B}) = 8$, n(A) = 12 and n(A ∩ B) = 5. (1m)

(iii) If A and B are two sets such that n(A) = 36 and n(B) = 55 and n(A ∩ B) = 30, then find n(A – B) (2m)

- OR -

If A and B are two sets such that n(A) = 36 and n(B) = 55 and n(A ∩ B) = 30, then find n(only B) (2m)

37 During examination, students make their time table and fix the study hours for a particular subject or fix the range of number of hours. They connect the number of hours with the outcome in the mathematical terms. Outcome is a function of qualitative use of number of hours. Let's consider a function $f = \left\{\left(x, \frac{1}{1-x^2}\right) : x \in R, x \neq \pm 1\right\}$ from R into R. Then answer the following

(i) Find the real number from co-domain which is associated with x = 0.1 (1m)

(ii) Find the Pre-image of $\frac{-1}{2}$. (1m)

(iii) Find the domain of the function *f* (2m)

- OR -

Find the range of the function *f* (2m)

38 The marks of four students out of 100 in 4 tests are given below and grading scheme is also given. Read the given information carefully and answer the following.

Name	***Test 1***	***Test 2***	***Test 3***	***Test 4***
Pranchi	85	93	94	89
Reshma	75	86	76	75
Ankit	92	83	44	60
Sunil	59	81	62	73

Grading System	
Average Marks (x)	***Grade***
$x \geq 91$	A_1
$90 \geq x \geq 81$	A_2
$80 \geq x \geq 71$	B_1
$70 \geq x \geq 61$	B_2
$60 \geq x \geq 51$	C

(i) To get a grade A_1, what will be the minimum marks Prachi should score in Test 5 (1m)

(ii) If Ankit scored 91 marks in his Test 5, then what will be his overall grade. (1m)

(iii) To get average marks more than Ankit, what will be the minimum marks Sunil have to score in Test 5 (2m)

- OR -

Reshma was not able to take Test 5 as she was ill. What will be Reshma's grade if the teacher gives her average of 4 test in the Test 5. (2m)

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ANSWERS

1	c) [-5, -3]	5	d) 4	9	a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$	13	b) $-i$
2	a) 0	6	d) $\frac{2\pi}{3}$	10	a) i	14	c) $-x > -5$
3	d) 10	7	d) $-\frac{1}{\sqrt{3}}$	11	c) 11	15	c) 86F & 95F
4	c) $\tan 2\theta$	8	b) $\frac{41}{360}\pi^c$	12	d) $3\sqrt{5}$	16	b) 4^5
17	c) 100						
18	a) 72						
19	(C) A is true but R is false						
20	(A) Both A and R are true and R is the correct explanation of A						

21 If $\tan\theta = \frac{1}{2}$ and $\tan\emptyset = \frac{1}{3}$, then find the value of $\theta + \emptyset$

(d), we know $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

So $\tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$

$\Rightarrow \tan(\theta+\phi) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}$

$= \frac{5/6}{5/6} = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \theta + \phi = \frac{\pi}{4}$

OR

Prove that $\cot x \cdot \cot 2x - \cot 2x \cdot \cot 3x - \cot 3x \cdot \cot x = 1$

$\text{L.H.S.} = \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$

$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$

$= \cot x \cot 2x - \cot(2x + x)(\cot 2x + \cot x)$

$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot 2x + \cot x}\right](\cot 2x + \cot x)$

$= \cot x \cot 2x - \cot 2x \cot x + 1$

$= 1$

22	Express $\frac{3-i}{5+6i}$ in the form of (a + *i*b) $\frac{(3-i)(5-6i)}{25+36} = \frac{15-18i-5i+6i^2}{61}$ $= \frac{9-23i}{61} = \frac{9}{61} - \frac{23}{61}i$	
23	Solve the following system of linear inequalities: $5x - 7 < 3(x + 3)$ $\quad 1 - \frac{3x}{2} \leq x - 4$ $X < 8$ $\quad 1 \leq x$ Solu: [2, 8)	
24	How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of the digits is not allowed? **The required number = $^6P_3 - {}^5P_2$ = $\frac{6!}{(6-3)!} - \frac{5!}{(5-2)!}$** **= $\frac{6!}{3!} - \frac{5!}{3!}$ = (4 x 5 x 6) – (4 x 5) = 100** OR In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers? required number of ways of selecting cricket team. $= {}^5C_4 \times {}^{12}C_7$ $= \frac{5!}{4!(5-4)!} \times \frac{12!}{7!(12-7)!} = \frac{5\times 4!}{4!\,1!} \times \frac{12\times 11\times 10\times 9\times 8\times 7!}{7!\,5!}$ $= 3960$	
25	Find the mean deviation about the median for the data: 2, 3, 5, 6, 8, 10, 12, 17, 20, 26 Median = 9 $\frac{1}{n}\sum\lvert xi - \bar{x}\rvert = \frac{1}{10}$ x 61 = 6.1	
26	If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 3, 5}, B = {2, 4, 6, 7} and C = {2, 3, 4, 8} then find (i) $(B \cup C)^I$ and (ii) $(C - A)^I$ (i) B ∪ C = {2,3,4,6,7,8} $\quad$ (ii) C – A = {4, 8} $(B \cup C)^I$ = {1,5,9,10} $\quad$ $(C - A)^I$ = {1, 2, 3, 5, 6, 7, 9, 10}	
27	Find the domain and range of the function $\sqrt{x^2 - 4}$ $D_f = (-\infty, -2] \cup [2, \infty)$ $\quad$ $R_f = [0, \infty)$	
28	Prove that cot 4x (sin 5x + sin 3x) = cot x (sin 5x − sin 3x) Given, L.H.S.= cot 4x (sin 5x + sin 3x) $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$	- OR - Evaluate $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi + x)\right]$ L.H.S.= $= \cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x)\right]$

$= \frac{\cos 4x}{\sin 4x}(2 \sin 4x \cos x)$

$= 2 \cos 4x \cos x$

R.H.S.$= \cot x(\sin 5x - \sin 3x)$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

R.H.S.$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$

$= \frac{\cos x}{\sin x}[2\cos 4x \sin x]$

$= 2\cos 4x \cos x$

L.H.S.=R.H.S.

$= \sin x \cos x [\tan x + \cot x]$

$= \sin x \cos x \left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right]$

$= \sin x \cos x \left[\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right]$

$= \sin^2 x + \cos^2 x$

$= 1$

29 Evaluate $(1 + i)^6 + (1 - i)^3$

$(1 + i)^6 = \{(1 + i)^2\}^3 = (1 + i^2 + 2i)^3$

$= (1 - 1 + 2i)^3 = 8i^3 = -8i$

$(1 - i)^3 = 1 - i^3 - 3i + 3i^2$

$= 1 + i - 3i - 3 = -2 - 2i$

$(1 + i)^6 + (1 - i)^3 = -8i - 2 - 2i = -2 - 10i$

- OR -

If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, then find $x + y$

Consider $\left(\frac{1+i}{1-i}\right) = \frac{(1+i)^2}{1^2 - i^2} = \frac{1 + i^2 + 2i}{2} = i.$

and $\frac{1-i}{1+i} = \frac{(1-i)^2}{1^2 - i^2} = -i$

$\therefore \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$

$\Rightarrow i^3 + i^3 = x + iy$

$\Rightarrow 2i^3 = x + iy$

$\Rightarrow -2i = x + iy$

$\Rightarrow x = 0, y = -2$

$\therefore x + y = -2$

30 Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

(i) do the words start with P

(ii) do all the vowels always occur together

(iii) do the vowels never occur together

(i) Let start with P

Then required number of arrangements = $\frac{11!}{4!\ x\ 3!\ x\ 2!}$

(ii) Total 5 Vowels in the word & Consider vowels as 1 unit

Then the arrangement of vowels = $\frac{5!}{4!}$

Then required number of arrangements = $\frac{8!}{3!\ x\ 2!} \times \frac{5!}{4!}$

(iii) The required number of arrangements =

= The total no of arrangements – The no of arrangements where all the vowels occur together.

$= \frac{12!}{4!\ x\ 3!\ x\ 2!} - \frac{8!}{3!\ x\ 2!} \times \frac{5!}{4!}$

OR

Find the Value of *'n'* such that ${}^{n}P_5 = 42.{}^{n}P_3$

$$\frac{n!}{(n-5)!} = 42 \text{ x } \frac{n!}{(n-3)!}$$

$$\frac{n\,x\,(n-1)\;x\;(n-2)\,x\;(n-3)\,x\;(n-4)\,x\;(n-5)!}{(n-5)!} = 42 \text{ x } \frac{n\,x\,(n-1)\;x\;(n-2)\,x\;(n-3)!}{(n-3)!}$$

$(n - 3\,(n - 4) = 42$

$n^2 - 7n - 30 = 0$

$(n - 10)\,(n + 3) = 0$

$n - 10 = 0$ or $n + 3 = 0$

$n = 10$ or $n = -3$

As n cannot be negative, so $n = 10$.

31 Find the mean deviation about the mean for the following data:

Class	*Mid value* (x_i)	*Frequency* (f_i)	$d_i = \frac{x_i - A}{h} = \frac{x_i - 35}{10}$	$f_i d_i$	$\lvert x_i - \bar{x} \rvert = \lvert x_i - 35 \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
0–10	5	4	–3	–12	30	120
10–20	15	6	–2	–12	20	120
20–30	25	10	–1	–10	10	100
30–40	35	20	0	0	0	0
40–50	45	10	1	10	10	100
50–60	55	6	2	12	20	120
60–70	65	4	3	12	30	120
		N = 60		$\Sigma f_i d_i = 0$		$\Sigma f_i \lvert x_i - \bar{x} \rvert = 680$

$$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{N} \text{ h}$$

$$= 35 + \frac{0}{60} \times 10 = 35$$

$$\text{M.D.} = \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - \bar{x} \rvert$$

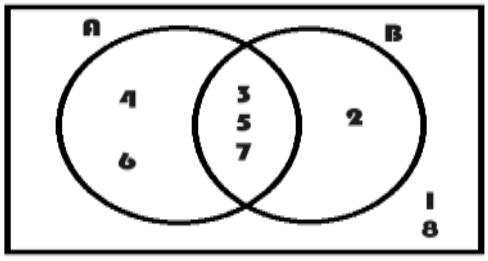
$$= \frac{1}{60} \times 680 = 11.33.$$

32 Let U = {x ∈ N : x ≤ 8}, A = {x ∈ N : $5 < x^2 < 50$},
B = {x ∈ N : x is a prime number less than 10}.

(vi) Draw a Venn Diagram to show the relationship between the given sets
(vii) list the elements of A^I
(viii) list the elements of B^I
(ix) list the elements of A – B
list the elements of $A \cap B^I$

U = {1,2,3,4,5,6,7,8}
A = {3,4,5,6,7}
B = {2,3,5,7}

(i)



(ii) list the elements of $A^I = \{1, 2, 8\}$

(iii) list the elements of $B^I = \{1, 4, 6, 8\}$

(iv) list the elements of $A - B = \{4, 6\}$

(v) list the elements of $A \cap B^I = \{3, 4, 5, 6, 7\} \cap \{1, 4, 6, 8\} = \{4, 6\}$

33 Prove that: $\cos 2x \cdot \cos\frac{x}{2} - \cos 3x \cdot \cos\frac{9x}{2} = \sin 5x \cdot \sin\frac{5x}{2}$

$$\text{L.H.S.} = \frac{1}{2}\left[2\cos 2x\cos\frac{x}{2} - 2\cos\frac{9x}{2}\cos 3x\right]$$

$$= \frac{1}{2}\left[\cos\left(2x+\frac{x}{2}\right)+\cos\left(2x-\frac{x}{2}\right)-\cos\left(\frac{9x}{2}+3x\right)-\cos\left(\frac{9x}{2}-3x\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{5x}{2}+\cos\frac{3x}{2}-\cos\frac{15x}{2}-\cos\frac{3x}{2}\right] = \frac{1}{2}\left[\cos\frac{5x}{2}-\cos\frac{15x}{2}\right]$$

$$= \frac{1}{2}\left[-2\sin\left\{\frac{\frac{5x}{2}+\frac{15x}{2}}{2}\right\}\sin\left\{\frac{\frac{5x}{2}-\frac{15x}{2}}{2}\right\}\right]$$

$$= -\sin 5x\ \sin\left(-\frac{5x}{2}\right) = \sin 5x\ \sin\frac{5x}{2} = \text{R.H.S.}$$

- OR -

Prove that:

$$2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$$

$$\text{Given, L.H.S.} = 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)$$

$$\left[\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$$

$$\Rightarrow 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right)\cos\left(-\frac{\pi}{13}\right)$$

$$\Rightarrow 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right)\cos\left(\frac{\pi}{13}\right)$$

$$\Rightarrow 2\cos\frac{\pi}{13}\left(\cos\frac{9\pi}{13} + \cos\left(\frac{4\pi}{13}\right)\right)$$

$$= 2\cos\frac{\pi}{13}\left(2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right)$$

$$= 4\cos\frac{\pi}{13}\left(\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{5\pi}{26}\right)\right)$$

$$= 4\cos\frac{\pi}{13}\left(0\times\cos\left(\frac{5\pi}{26}\right)\right)$$

$$= 0$$

$$= \text{R.H.S.}$$

34 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(iv) no girl?

(v) at least one boy and one girl?

(vi) at least 3 girls?

(i) Since, the team will not include any girl, therefore, only boys are to be selected.
5 boys out of 7 boys can be selected in ${}^7C_5$ ways.
Therefore, the required number of ways $= {}^7C_5 = \frac{7!}{5!\ (7-5)!} = 21$

(ii) Since, at least one boy and one girl are to be there in every team.
(a) 1 boy and 4 girls ${}^7C_1 \cdot {}^4C_4$ ways.
(b) 2 boys and 3 girls ${}^7C_2 \cdot {}^4C_3$ ways.
(c) 3 boys and 2 girls ${}^7C_3 \cdot {}^4C_2$ ways
(d) 4 boys and 1 girl. ${}^7C_4 \cdot {}^4C_1$ ways
Therefore, the required number of ways $= {}^7C_1 \cdot {}^4C_4 + {}^7C_2 \cdot {}^4C_3 + {}^7C_3 \cdot {}^4C_2 + {}^7C_4 \cdot {}^4C_1$
$= 7 + 84 + 210 + 140 = 441$

(iii) Since, the team has to consist of at least 3 girls, the team can consist of
(a) 3 girls and 2 boys, ${}^4C_3 \cdot {}^7C_2$ ways.
(b) 4 girls and 1 boy. ${}^4C_4 \cdot {}^7C_1$ ways
Therefore, the required number of ways $= {}^4C_3 \cdot {}^7C_2 + {}^4C_4 \cdot {}^7C_1$
$= 84 + 7 = 91$

OR

What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these

(vi) four cards are of the same suit
(vii) four cards belong to four different suits
(viii) are face cards
(ix) two are red cards and two are black cards
(x) cards are of the same colour?

(iii) are face cards $= {}^{12}C_4$
$= \frac{12!}{4!.8!} = 495$

(i) four cards are of the same suit
$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$
$= 4 \times {}^{13}C_4$

(iv) two are red cards and two are black cards $= {}^{26}C_2 \times {}^{26}C_2$
$= \frac{26!}{2!.24!}$

(ii) four cards belong to four different suits,
$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$
$= 13^4$

(v) cards are of the same colour?
$= {}^{26}C_4 + {}^{26}C_4$
$= 2 \times \frac{26!}{4!.22!}$

35	The diameters of circles (in mm) drawn in a design are given below. Calculate Mean, Variance and Standard Deviation for the data

Class	Closed class	Freq. (f_i)	Mid value (x_i)	$y_i = \frac{x_i - A}{h} = \frac{x_i - 42.5}{4}$	$f_i y_i$	y_i^2	$f_i y_i^2$
33–36	32.5–36.5	15	34.5	–2	–30	4	60
37–40	36.5–40.5	17	38.5	–1	–17	1	17
41–44	40.5–44.5	21	42.5	0	0	0	0
45–48	44.5–48.5	22	46.5	1	22	1	22
49–52	48.5–52.5	25	50.5	2	50	4	100
		N = 100			$\Sigma f_i y_i = 25$		$\Sigma f_i y_i^2 = 199$

$$\text{Mean} = A + \frac{\Sigma f_i y_i}{N} \times h$$

$$= 42.5 + \frac{25}{100} \times 4 =$$

$$= 42.5 + 1 = 43.5$$

Variance (σ^2) =

$$= \left[\frac{\Sigma f_i y_i^2}{N} - \left(\frac{(\Sigma f_i y_i)}{N}\right)^2\right] h^2$$

$$= \left[\frac{199}{100} - \left(\frac{25}{100}\right)^2\right] 16$$

$$= \left[1.99 - \frac{1}{16}\right] 16 = 30.84$$

$$(\sigma) = \sqrt{\text{Var}(X)} = \sqrt{30.84}$$

$$= 5.55 \text{ mm}$$

36 (i) A and B are two sets such that n(A – B) = 20 + x, n(B – A) = 3x and n(A ∩ B) = x + 1. If n(A) = n(B) then find 'x'. (1m)

An: x = 10

(ii) Check whether the following statement is True or False with reason:
A and B are two sets such that $n(A \cap \bar{B}) = 8$, n(A) = 12 and n(A ∩ B) = 5. (1m)

An: False, $n(A \cap \bar{B})$ = n(A) – n(A ∩ B)

(iii) If A and B are two sets such that n(A) = 36 and n(B) = 55 and n(A ∩ B) = 30, then find n(A – B) (2m)

An: n(A – B) = n(A) – n(A ∩ B) = 36 – 30 = 6

- OR -

If A and B are two sets such that n(A) = 36 and n(B) = 55 and n(A ∩ B) = 30, then find n(only B)

n(only B) = n(B- A) = n(B) – n(A ∩ B) = 55 – 30 = 25

37 (i) Find the real number from co-domain which is associated with x = 0.1 is $\frac{100}{99}$

(ii) Find the Pre-image of $\frac{-1}{2}$ in the co-domain. x = $\pm\sqrt{3}$

(iii) Find the domain of the function f R – {-1, 1}

- OR -

Find the range of the function f R – [0, 1)

$1 - x^2 = 0$

$1 = x^2$

$x = \sqrt{1}$

$x = \pm 1$

So the domain of function is $R - \{\pm 1\}$

$\frac{1}{1-x^2} = y$

$1 = y - yx^2$

$yx^2 = y - 1$

$x^2 = \frac{y-1}{y}$

$x = \sqrt{\frac{y-1}{y}}$

So the Range is $R - [0, 1)$

38 The marks of four students out of 100 in 4 tests are given below and grading scheme is also given. Read the given information carefully and answer the following.

Name	***Test 1***	***Test 2***	***Test 3***	***Test 4***
Pranchi	85	93	94	89
Reshma	75	86	76	75
Ankit	92	83	44	60
Sunil	59	81	62	73

Grading System	
Average Marks (x)	***Grade***
$x \geq 91$	A_1
$90 \geq x \geq 81$	A_2
$80 \geq x \geq 71$	B_1
$70 \geq x \geq 61$	B_2
$60 \geq x \geq 51$	C

38 (i) To get a grade A_1, what will be the minimum marks Prachi should score in Test 5 (1m)

Let Prachi marks in test-5 be y

then $\frac{85 + 93 + 94 + 89 + y}{5} \geq 91$

$y \geq 455 - 361$

$y \geq 94$

Minimum marks Prachi should get = 94

(ii) If Ankit scored 91 marks in his Test 5, then what will be his overall grade. (1m)

Average marks of Ankit

$= \frac{92 + 83 + 44 + 60 + 91}{5}$

$= \frac{370}{5} = 74$

So grade of Ankit = B_1

(iii) To get average marks more than Ankit, what will be the minimum marks Sunil have to score in Test 5 (2m)

Let marks of Sunil in test-5 be y

then $\frac{59+81+62+73+y}{5} > 74$

$y > 370 - 275$

$y > 95$

Minimum marks, $y = 96$

- OR -

Reshma was not able to take Test 5 as she was ill. What will be Reshma's grade if the teacher gives her average of 4 test in the Test 5. (2m)

Average marks of Reshma in 4 test

$= \frac{75+86+76+75}{4} = 78$

Marks in test 5 = 78

Average marks in 5 test

$= \frac{75+86+76+75+78}{5} = 78$

Grade = B_1

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